§ 4. Excursus on Natural Theology
Lecture 9
Defense of the Second Premise of the *Kalam* Cosmological Argument

Last week we began talking about the *kalam* cosmological argument. I offered a defense of the first premise. Today we want to turn to the second premise of that argument which is that the universe began to exist.

This is obviously the more controversial of the two premises. It is fairly obvious, I think, that if the universe began to exist then the universe has a cause of its existence. But it is by no means obvious that the universe began to exist. So I want to examine both philosophical arguments and scientific evidence in support of this second premise.

If you were to ask me what the relationship is between these two, I would say that, for me at least, the first line of defense for this second premise is the philosophical arguments. I see the scientific evidence as simply a confirmation (empirically) of a conclusion already established on the basis of philosophical arguments. I will often speak of the support for this premise in terms of philosophical arguments and scientific confirmation.

Let's look at the first philosophical argument. Al-Ghazali, the 12th century Muslim theologian whom we've taken as our springboard for examining this argument, argued that if the universe never began to exist then there has been an infinite number of past events prior to today. But, he argued, an infinite number of things cannot exist. Therefore it follows that there cannot have been an infinite past. Al-Ghazali recognized that a *potentially* infinite number of things could exist, but what he denied was that an *actually* infinite number of things could exist. It is important that we understand this absolutely crucial distinction between the potential infinite and the actual infinite.

When we say that something is potentially infinite, we mean that something is indefinite but progressing toward infinity as an ideal limit which is never reached. You never actually arrive at infinity. Infinity is simply a limit concept which you approximate toward. For example, take any finite distance. You could divide that distance in half, and then into fourths, and then into eighths, and then into sixteenths, and then into thirty-secondths, ad infinitum. But you would never reach an "infinitieth" division. The number of divisions is potentially infinite in that you could go on dividing endlessly. But you never arrive at infinity. You would never have an actually infinite number of divisions or of parts. The symbol for this kind of infinity is the lemniscate or the lazy-eight (∞). This is the type of infinity that is used in calculus in mathematics where you have infinite limits.

By contrast with that, the actual infinite is an infinite which is, as it were, complete. The number of items in the collection is not growing toward infinity; it *is* infinite! It is complete and static and involves an actually infinite number of things.

This type of infinity is symbolized by the Hebrew letter *aleph* ($\aleph$) and is used in set theory. In set theory, mathematicians talk about sets like the set of natural numbers which have an actually infinite number of members in the set. The collection is not growing toward infinity as a limit. It is infinity. There are an actually infinite number of natural numbers in this set. $\aleph$ is a number. If you were to ask what is the number of elements in the set of natural numbers, the answer would be *aleph*-null ($\aleph_0$). That is the number of members in the set of natural numbers.[1]

Technically speaking, what defines a collection or a set as actually infinite is that it has a proper part which has the same number of members as the whole collection. So, for example, think about this. The number of odd numbers is the same as the number of all the natural numbers - namely, $\aleph_0$. It is exactly the same. There are just as many odd numbers as there are natural numbers, even though the natural numbers includes not only the odd numbers but an infinite number of even numbers as well! Technically speaking, the definition for an actual infinite is that it is a collection that has a proper part with the same number of members in it as the whole collection.

What al-Ghazali is claiming is that while potentially infinite collections can exist (that is to say, collections that are always finite at any point in time but they are growing toward infinity as a limit) there cannot be a collection that is actually infinite – that has an actually infinite number of members in it.

START DISCUSSION

Student: You indicated that the odd numbers would be equal to the actual number of numbers. It seems to me it is just half of the odd and even numbers.

Dr. Craig: Yes, and I am glad that you feel that way because this is precisely what will engender various absurd situations when you translate this out of the mathematical realm into the real world of people, and sticks, and rocks, and eggs, and things like that. You get extremely bizarre results precisely because of this. What al-Ghazali would say is while you could talk about actually infinite collections and do these mathematics on paper, it is not something that can exist in the real world because it will involve these sorts of counter-intuitive absurdities.

Student: If you have an infinite number of odd numbers and an infinite number of total numbers then they are both infinite and therefore the same number.

Dr. Craig: That is correct.

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Student: The potential infinite – that would be a concept where you could imagine something indefinite.

Dr. Craig: Yes.

Student: Therefore it is potential, but it is a concept of infinity rather than an actual number of items.

Dr. Craig: Yes! Very good! I am so impressed. Notice the distinction that she saw. $\aleph_0$ is a number. It is a quantity. It is the number of members in this set. The lemniscate (∞), or the potential infinite, isn't a number. It is a limit. It is an ideal limit concept, but it is not a number. That is important to understand.

Student: I know the lazy-eight (∞) is not a number, as you said, and you can't do math on it.

Dr. Craig: You can do calculus with it. It is used in calculus.

Student: OK, right. But you can't multiply it by two because then it is equal to itself. But in what way is the *aleph* ($\aleph$) a number because you can't do math on that either?

Dr. Craig: Actually you can. This is the interesting thing with these *alephs*, because there really is more than one of them. Remember I said the number of natural numbers is $\aleph_0$? But there are more real numbers than there are natural numbers.[2] You begin to get a whole series of these *alephs* that have subscripts. $\aleph_0$, $\aleph_1$, $\aleph_2$, and it goes to infinity. There are actually an infinite number of these infinites. This is where it just becomes completely beyond the human mind to comprehend. And you can do mathematical operations with these numbers. This is called transfinite arithmetic. For example, what is $\aleph_0+\aleph_0$? Well, the answer is $\aleph_0$. You can do transfinite arithmetic using these numbers. You can do multiplication, and you can do addition, and you can do exponentiation (like $\aleph_0$ to the second power, for example). This is a number that can be manipulated in arithmetic in this way.

What is interesting, and this will become significant when we talk about whether actual infinites can really exist, is that you can't do inverse operations like subtraction and division with them, because then you get self-contradictions. It is stipulated – it is part of the rules – that all you can do are these positive operations like addition and multiplication, and you can't do subtraction and division (that is prohibited).

Student: [off-mic] So one-over-*aleph* ($1/\aleph$) isn't zero?

Dr. Craig: Right, you can't do division with these sorts of things.

Student: You had said that you can have an infinite number of *alephs*. Would that be *aleph*-to-the-*aleph* ($\aleph^{\aleph}$)?

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Dr. Craig: That's a good question. I think that the number of *alephs* is $\aleph_0$ because they are enumerated by the natural numbers 0, 1, 2, 3, 4, ... So if you have them subscripted with the natural numbers, the number of *alephs* would be $\aleph_0$.

Student: Is it an actual infinite of *alephs*?

Dr. Craig: Yes. Right.

Student: Isn't the ratio of $\aleph_n$ divided by $\aleph_{n+1}$ always zero by definition because you can't put them into a one-to-one relationship?

Dr. Craig: You can't do those kind of inverse operations. That is prohibited. You are trying to do division with these, and you can't do that.

Student: I think you can prove that it is zero. I think that is known. I could be wrong.

Dr. Craig: So far as I know, you can't do those kinds of inverse operations like dividing one *aleph* by another.

Student: Speaking to the size of it – the next one is always . . .

Dr. Craig: Bigger. That is right. These are different sizes of infinites. That is correct. The $\aleph_1$ is a larger collection than $\aleph_0$. In that sense, it has members in it that the other one doesn't have.

END DISCUSSION

Al-Ghazali, as I said, has no problem with the idea of merely potential infinites. These are just ideal limits. But he argued that if an actually infinite number of things could exist then various absurdities would result. If we are to avoid these absurdities we have to deny that an actually infinite number of things can exist. That would imply that the number of past events in the history of the universe therefore cannot be actually infinite. It must be finite. Therefore, the universe cannot be beginningless. The universe must have begun to exist.

START DISCUSSION

Student: What are the absurdities?

Dr. Craig: I'll go into those in a moment. Of course, of course. I just wanted you to make sure we are all tracking together and I don't get ahead of you. You understand the basic argument; namely, it is absurd that an actually infinite number of things could exist, but a beginningless past would involve an actually infinite number of things; namely, past events. So the past can't be actually infinite. It must be finite, and therefore have a beginning. That is the basic argument that I want to make sure we all get.

Student: I am one of those that doesn't quite understand.[3] To me, the better way to understand what the term actual infinite means – what you are trying to do with saying actual is not infinite. Rather than say infinite, you can say finite. It seems like what you are saying is not-infinite means it is finite. It seems like we are starting with a contradiction of terms to begin with.

Dr. Craig: That is why it is important to make sure we are on the same page with the definitions.

Student: I'm not there yet.

Dr. Craig: When mathematicians talk about an actual infinite, as I say, they don't mean what you just said – that this is some kind of finite thing or contradiction. It means that the collection is complete, it is not growing toward infinity as a limit. There is a real infinite number of things in that collection. That is the force of the word "actual."

Student: And that is a contradiction in terms.

Dr. Craig: Well, that is very interesting. Is it? I am going to speak to that in a minute. If you follow the rules and the conventions laid down by set theory, you won't run into any contradictions. It isn't logically contradictory *if you follow the rules and obey the axioms and conventions.* But there is the rub. I will say something about that in a minute.

Student: So you are saying you can add it and multiply it, but you can't reverse it?

Dr. Craig: Yes.

Student: I'm not sure I follow how you can add it, multiply it, but then you can't reverse the process back to the original. You can you can add and multiply actual infinity, but you couldn't divide it or subtract it. So you can't reverse it. I don't understand how you can't go back to the original.

Dr. Craig: You can't mathematically. The difficulty is, I think, that if this is something that really exists (like it is a bunch of eggs or coins or people), you could! I think that illustrates what we are saying. While this works on paper (if you obey the rules), there is no reason to think that those sort of rules hold in reality, and you are going to get into difficulties. I have yet to illustrate these.

END DISCUSSION

It is very frequently alleged that al-Ghazali's sort of argument is invalidated by modern mathematics. In modern set theory, as I've said, the use of actually infinite sets is commonplace. The number of members in the set of the natural numbers is actually infinite, not just potentially infinite. Many people have inferred that given the coherence of infinite set theory in mathematics that this sort of argument is just a non-starter.

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But is that really the case? Modern set theory shows that if you adopt certain axioms and rules, then you can talk about actually infinite collections in a consistent way, without contradicting yourself, as I said in response to an earlier question. All this does is succeed in setting up a certain universe of discourse for talking consistently about actual infinites. But it does absolutely nothing to show that such mathematical entities really exist or that an actually infinite number of things can really exist. If Ghazali is right, this universe of discourse may be regarded simply as a fictional realm, rather like the world of Sherlock Holmes in the Arthur Conan Doyle novels, not something that exists in the real world.

START DISCUSSION

Student: Couldn't you also criticize the criticism by applying Gödel's Incompleteness Theorem – in any mathematical system, you are going to assume something is true in order for it to work, and you can't prove the assumption is true?

Dr. Craig: I don't think that that is relevant to the concern that we are raising here. I think it would mean you couldn't prove the consistency of infinite set theory, but we are not trying to do that. So I don't think that that result is pertinent to the question that we are raising here.[4]

Student: The adding and multiplying of the *alephs* is possible because they are both infinite. But taking from it would obviously make it a part of an infinite, which doesn't exist, which proves the point that having every odd number equals the same amount of every other number also can't exist, so no actual infinity actually exists, except for possibly God. The only infinity – the real infinity that never had a beginning and never has an end – that always counts as infinity is just God. Is that right?

Dr. Craig: You raised a number of questions there. The reason that you can't do these inverse operations in transfinite arithmetic is because you get self-contradictory results. Let me give an example. Suppose you take the natural numbers and you subtract all the odd numbers. How many numbers are left over? All the even numbers, right? So infinity minus infinity is infinity. But suppose instead you subtract from the natural numbers all the numbers greater than 2. Now how many are left over? Well, 3! So infinity minus infinity is 3. In fact, you can get any answer to infinity minus infinity from zero to infinity. As I say, there is no well defined result for the equation infinity minus infinity equals blank. You can get any answer from zero to infinity. You get self-contradictory answers. So these operations are simply prohibited to the mathematician.

With respect to God, people will often ask this question: "But isn't God infinite?" Here I think it is very important to understand that the infinity of God is not a quantitative concept. God is not a mathematical quantity. The infinity of God is not the infinity of a collection that is made up of an infinite number of definite and discrete parts. When

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theologians talk about God as infinite, it is more, as it were, a qualitative infinite, not a quantitative infinite. That is to say, God is omnipotent, omniscient, morally perfect, eternal, necessary, all-loving. Those aren't quantitative concepts. Indeed, in a sense there isn't any separate attribute of God called "infinity." It is kind of just an umbrella term for all of his superlative attributes. If you were to take away in your mind omniscience, omnipotence, eternity, necessity, holiness, there wouldn't be any attribute left over called "infinity." That is just an umbrella term for all those superlative attributes that God possesses. So we shouldn't think of God's infinity as a quantitative concept. He doesn't involve an actually infinite number of definite and discrete pieces that go to make up his being.

Student: About God's infinity – is it possible that with God he may be able to understand and do calculations with an actual infinite? For example, might God have considered an actually infinite number of counterfactuals before creating this universe?

Dr. Craig: Wow. OK. You are getting into very difficult issues of metaphysics now. What you are raising is this old problem that we've encountered again and again, and that is do abstract objects exist. Because propositions (or counterfactuals) would be examples of abstract objects. If there are abstract objects like mathematical objects, numbers, propositions, possible worlds, properties, then these are plausibly actually infinite. But I'm persuaded that these things don't exist and that therefore they do not contradict al-Ghazali's statement that there cannot be an actually infinite number of things. The anti-realist isn't bothered by those sorts of counter-examples. In order for that to be an effective counter-example to al-Ghazali, you would need a proof that Platonism is true, and there isn't any such proof.[5] Platonism is just one alternative among many, and it is not incumbent upon us. We really get into the deep weeds when we start talking about these things!

END DISCUSSION

The way in which al-Ghazali brings out the real impossibility of an actually infinite number of things is by imagining what it would be like if such a collection could exist and then drawing out the absurd consequences of it. Let me share with you one of my favorite illustrations called "Hilbert's Hotel," which is the brainchild of the great German mathematician David Hilbert.

Hilbert warms up by inviting us to imagine an ordinary hotel with a finite number of rooms. Let's suppose that the rooms are completely occupied. There is not a single vacant room throughout the entire hotel. Now suppose a new guest shows up at the front desk asking for a room. "Sorry," the manager says, "All the rooms are occupied," and the new guest has to be turned away.

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But now, Hilbert imagines, let's suppose we've got a hotel with an infinite number of rooms, and let's suppose once again that the hotel is completely occupied. We have to fully appreciate this fact. There is not a single vacancy in the entire infinite hotel; every room has a flesh-and-blood person in it. Now suppose a new guest shows up at the front desk, asking for a room. "No problem," says the manager. He moves the guest that was in room #1 into room #2, he takes the guest that was in room #2 and puts him in room #3, he takes the guest that was in room #3 and puts him in room #4, out to infinity. As a result of these transpositions, room #1 now becomes vacant, and the new guest is easily accommodated. And yet, before he arrived, all the rooms were already full!

It gets even worse! Now, Hilbert says, let's imagine that an infinite number of new guests shows up at the front desk asking for rooms. "No problem, no problem!" says the manager. He moves the person who was in room #1 into room #2, the person who was in room #2 into room #4, the person who was in room #3 into room #6. He puts each person into the room number double his own. Since any number multiplied by two is always an even number, that means all the guests wind up in the even-numbered rooms. As a result, all the odd-numbered rooms become vacant, and the infinity of new guests gratefully checks in. And yet, before they arrived, all the rooms were already full!

As one student remarked to me, Hilbert's Hotel, if it could exist, would have to have a sign outside: "No Vacancy (Guests Welcome)."

Can such a hotel exist in reality? Since nothing hangs on the illustrations involving a hotel, this argument can be generalized to show that the existence of an actually infinite number of things is really absurd.

I hadn't planned on sharing further difficulties with Hilbert's Hotel, but given that it has already come up, let me say that the German mathematician didn't even fully demonstrate the absurdity of this hotel. Because he never asked: what would happen if people started checking out of the hotel? Suppose all the people in the odd-numbered rooms check out – 1, 3, 5, 7, and so forth. How many guests are left? Well, all the even-numbered guests. An infinite number of guests are still left in the hotel even though an equal number has already checked out and left the hotel. But now let's suppose instead that all of the guests in the rooms 3, 4, 5, 6, 7, out to infinity checked out. How many guests are left now? If there is a room #0, just three are left. Yet, the same number of guests checked out this time as when all of the odd-numbered guests left. You subtract identical quantities from identical quantities and you get non-identical results, which is absurd.[6]

Someone might say that you can't do inverse operations with mathematical quantities. Not on paper perhaps, but there is no way you can stop people from checking out of a

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real hotel. If you try to bar the door, they will go out the windows. This illustrates the absurdity of the real existence of an actually infinite number of things.

Sometimes students will react to Hilbert's Hotel by saying that these absurdities result because the concept of infinity is just beyond us and we don't understand it. But that reaction is mistaken and naïve. As I said, infinite set theory is a highly developed and well-understood branch of modern mathematics. These absurdities result because we *do* understand the nature of the actual infinite. Hilbert was a smart man, and he well knew how to illustrate the bizarre consequences of the existence of an actually infinite number of things.

START DISCUSSION

Student: I taught this argument at Mount Vernon Presbyterian School to a bunch of high schoolers. We got to this philosophical understanding of actual infinite. I used the example that you gave in Lee Strobel's *A Case for a Creator* with marbles. If you have an infinite number of marbles and you want to give another person an infinite number of marbles you can do it in different ways, and you would get absurd results. I just wanted to say that they really enjoyed talking about it and they understood it. So anyone who says they can't understand these things, high schoolers can really get into this kind of stuff. They just really enjoyed it. I just wanted to say that.

Dr. Craig: Thanks for the encouraging words.

Student: You stated that infinity in mathematics . . . the reason they are not allowed to do all of these sort of subtraction, etc. is because you get contradictions. But they say as long as you don't do that it is not contradictory. It reminds me the point that Wes Morriston likes to bring up that the contradictions arise when you move people around whereas the past (which is what you are trying to argue is finite) isn't something . . . you can't move around past dates like you can people in a hotel or coins or marbles.

Dr. Craig: I've never understood why someone thinks that that is a good objection. We are obviously not talking with regard to Hilbert's Hotel about a real hotel that is built out of bricks and wood and has people trying to walk down infinite hallways to get out the door. It is a conceptual thought experiment. You imagine the hotel with all the people in the rooms and then, as it were, in thought just eliminate all the people in the odd-numbered rooms. Just vaporize them or something. Then you've got all the people left in the even-numbered rooms. You don't want to get into difficulties about physically moving them about and so forth. Similarly, with respect to the number of past events. If you imagine the number of past days in the history of the universe, it is easy to just mentally annihilate every other day, or all the odd-numbered days, and ask how many are left over. The answer is obvious. There would still be all the even-numbered days, which is the same number. It seems to me that that kind of objection just fails to reckon with the nature of a thought experiment which isn't based upon real, physical movements and operations.

Student: It seems a lot of the more knee-jerk reaction that some people will have (mostly atheists) will say something like, *OK, there is no absurdity. That is just what happens when you have infinity. That is just the way infinity works, and there is no problem to it.*

Dr. Craig: OK. Thank you for saying that, because that is the segue to the next point.

END DISCUSSION

Really, the only thing the critic can do at this point is to just bite the bullet and say that Hilbert's Hotel is not absurd. *Yeah, that's right; that is the way it would be.* Sometimes they will justify this by saying that if an actual infinite could exist then such situations are exactly what we should expect. But, again, I don't think this is an adequate response. Hilbert would, of course, agree that *if* an infinite hotel could exist then the situation that he has imagined is what we would expect. Otherwise, it wouldn't be a good illustration! Right? So, of course this is what would happen if an actually infinite number of things could exist.[7] But the question is: is such a hotel really possible? I think that these illustrations show that, no, such a thing is not really possible. It is metaphysically absurd.

So I think al-Ghazali's first argument is a good one. It shows that the number of past events must be finite. Therefore, the universe must have had a beginning.

We can summarize this argument as follows:

1. An actual infinite cannot exist.

2. An infinite temporal regress of events is an actual infinite.

3. Therefore, an infinite temporal regress of events cannot exist.

Next time we will look at the second independent argument that al-Ghazali offers for the beginning of the universe and the finitude of the past.[8]

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8 Total Running Time: 36:22 (Copyright © 2015 William Lane Craig)