

## THE APPLICABILITY OF MATHEMATICS

One of the burning questions in the philosophy of mathematics concerns mathematics' applicability to physical phenomena. Applicability has to do with what mathematician and physicist Eugene Wigner famously called "the unreasonable effectiveness of mathematics in the natural sciences."<sup>1</sup> Mathematics is the language of nature. That is to say, the laws of nature may be expressed as mathematical equations which describe the phenomena to an astonishing degree of accuracy.

In his seminal paper Wigner's central point that Mathematical concepts turn up in "entirely unexpected connections" in physics and often permit "an unexpectedly close and accurate description" of the phenomena in these connections.<sup>2</sup>

In unfolding his claim, Wigner explores the role of mathematics in physics and why mathematics' success in that role appears "so baffling." With respect to mathematics' role in physics, Wigner notes that while mathematics is useful in physics for evaluating the consequences of the laws of nature, a role which he associates with applied mathematics, it also plays a more "important" and "sovereign" role in physics, namely, to enable the formulation of the laws of nature themselves in the language of mathematics in order to be an apt object for the use of applied mathematics.

Wigner notes that "the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena."<sup>3</sup> He provides three examples in support: Newton's second law of motion, elementary

quantum mechanics, and quantum electrodynamics. Wigner takes his examples—“which could be multiplied almost indefinitely”—to illustrate the “appropriateness” and “almost fantastic accuracy” of the mathematical formulation of the laws of nature.

With respect to why mathematics’ role in physics appears “so baffling,” Wigner notes that his three examples represent an increasing independence of empirical experience in favor of reliance on mathematical concepts which are chosen for aesthetic reasons rather than empirical applicability:

whereas it is unquestionably true that the concepts of elementary mathematics and particularly elementary geometry were formulated to describe entities which are directly suggested by the actual world, the same does not seem to be true of the more advanced concepts, in particular the concepts which play such an important role in physics. . . . Most more advanced mathematical concepts, such as complex numbers, algebras, linear operators, Borel sets - and this list could be continued almost indefinitely - were so devised that they are apt subjects on which the mathematician can demonstrate his ingenuity and sense of formal beauty.<sup>4</sup>

Wigner’s point is well-taken. As philosopher of mathematics Penelope Maddy emphasizes, mathematicians employ what she calls “maximizing principles of a sort quite unlike anything that turns up in the practice of natural science: crudely, the scientist posits only those entities without which she cannot account for our observations, while the set theorist posits as many entities as she can, short of inconsistency.”<sup>5</sup> Maddy

identifies quite a few of these “rules of thumb” followed by mathematicians in choosing their axioms and constructing their theories, such as *maximize, richness, diversity, one step back from disaster, etc.*<sup>6</sup> Similarly Wigner observes, “The great mathematician fully, almost ruthlessly, exploits the domain of permissible reasoning and skirts the impermissible.”<sup>7</sup>

The “principal point” which is relevant to the uncanny effectiveness of mathematics is that mathematicians are free to define new concepts with a view, not of scientific utility, but of “permitting ingenious logical operations which appeal to our aesthetic sense both as operations and also in their results of great generality and simplicity.”<sup>8</sup> Wigner is not talking about aesthetics in the artistic sense, but in the sense of mathematical beauty, what Maddy calls mathematical depth. Mathematics is an *a priori* discipline which is independent of the physical world.

Moreover, when we reflect that mathematical objects, even if they exist, are causally effete, it is surprising that such objects should be significantly effective in physics.<sup>9</sup> Wigner muses, “It is difficult to avoid the impression that a miracle confronts us here.”<sup>10</sup>

Accordingly, the following seems to be a faithful formulation of Wigner’s argument:

1. Mathematical concepts arise from the aesthetic impulse in humans and have no causal connection to the physical world.

2. It would be surprising to find that what arises from the aesthetic impulse in humans and has no causal connection to the physical world should be significantly effective in physics.

3. Therefore, it would be surprising to find that mathematical concepts should be significantly effective in physics.

4. The laws of nature can be formulated as mathematical descriptions (concepts) which are often significantly effective in physics.

5. Therefore, it is surprising that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.

Given that something surprising merits *prima facie* an explanation, we wonder what might be the explanation of the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.

Wigner, despite his characterization of the applicability of mathematics to the physical world as a miracle, in the end regarded it as a mystery. He concluded, “The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.”<sup>11</sup> Wigner never actually considered in his essay whether the applicability of mathematics might be a literal miracle, so that theism furnishes a good explanation of mathematics’ applicability. He considered at most naturalistic explanations of it and, finding none to be satisfactory, therefore

concluded “that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it.”<sup>12</sup>

But suppose we take the theistic hypothesis seriously. Theists will have a considerably easier time, I think, explaining the applicability of mathematics than will naturalists. Theists hold that there is a personal, transcendent being (a.k.a. God) who is the Creator and Designer of the universe. Naturalists hold that all that exists concretely is space-time and its physical contents. Now whether one is a realist or an anti-realist about mathematical objects, it appears that the theist enjoys a considerable advantage over the naturalist in explaining the uncanny success of mathematics.

Consider first realism’s take on the applicability of mathematics to the world. For the *non-theistic* realist, the fact that physical reality behaves in accord with the dictates of acausal mathematical entities existing beyond space and time is, in the words of philosopher of mathematics Mary Leng, “a happy coincidence.”<sup>13</sup> For consider: If, *per impossibile*, all the abstract objects in the mathematical realm were to disappear overnight, there would be no effect on the physical world. This is simply to underscore the fact that abstract objects are causally inert. The idea that realism somehow accounts for the applicability of mathematics “is actually very counterintuitive,” muses Mark Balaguer. “The idea here is that in order to believe that the physical world has the nature that empirical science assigns to it, I have to believe that there are causally inert mathematical objects, existing outside of spacetime,” an idea which is inherently implausible.<sup>14</sup>

By contrast, the *theistic* realist can argue that God has fashioned the world on the structure of the mathematical objects He has chosen. This is essentially the view that Plato defended in his dialogue *Timaieus*. Plato draws a fundamental distinction between the realm of static being (that which ever is) and the realm of temporal becoming (that which is ever becoming). The former realm is to be grasped by the intellect, whereas the latter is perceived by the senses. The realm of becoming is comprised primarily of physical objects, while the static realm of being is comprised of logical and mathematical objects. God looks to the realm of mathematical objects and models the world on it. The world has its mathematical structure as a result. Thus, the realist who is a theist has a considerable advantage over the naturalistic realist in explaining why mathematics is so effective in describing the physical world.

Now consider anti-realism of a *non-theistic* sort. Mary Leng says that on anti-realism relations which are said to obtain among pretended mathematical objects just mirror the relations obtaining among things in the world, so that there is no happy coincidence. Philosopher of physics Tim Maudlin muses, “The deep question of why a given mathematical object should be an effective tool for representing physical structure admits of at least one clear answer: because the physical world literally has the mathematical structure; the physical world is, in a certain sense, a mathematical object.”<sup>15</sup>

Well and good, but what remains wanting on naturalistic anti-realism is an explanation *why* the physical world should exhibit so elegant and stunning a mathematical structure in the first place. Not only so, but by choosing examples like the infinite-dimensional Hilbert space and

complex numbers, Wigner implicitly precluded the explanation that physical reality possesses such mathematical structures, since these cannot be physically realized in the universe. Mark Steiner provides numerous examples of the applicability of mathematical concepts that cannot be physically instantiated.<sup>16</sup> Some of his examples are the same ones to which Wigner already appealed, such as the descriptive applicability of the Hilbert space formalism to quantum mechanics, which Steiner calls “physically unintelligible.” So even if the physical universe had to have some mathematical structure, that fails to address the question raised by Wigner.

By contrast, the *theistic* anti-realist has a ready explanation of the applicability of mathematics to the physical world: God has created it according to a certain blueprint which He had in mind. There are any number of blueprints He might have chosen. On theistic anti-realism the laws of nature have the mathematical form they do because God has chosen to create the world according to the abstract model He had in mind. This was the view of the first century Jewish philosopher Philo of Alexandria, who maintained in his treatise *On the Creation of the World* that God created the physical world on the mental model in His mind. For a Jewish monotheist like Philo, the realm of Ideas does not exist, as Plato thought, independently of God but as the contents of His mind. Philo referred to the mind of God as God’s Logos (Word). The sensible world (*kosmos oratos*) is made on the model of the conceptual or intelligible world (*kosmos noētos*) that pre-exists in the Logos. Just as the ideal architectural plan of a city exists only in the mind of the architect, so the world of ideas exists solely in the mind of God.

Thus, the theist—whether he be a realist or an anti-realist about mathematical objects—has the explanatory resources to account for the otherwise unreasonable effectiveness of mathematics in physical science—resources which the naturalist lacks.

We may thus extend Wigner's argument:

1. Mathematical concepts arise from the aesthetic impulse in humans and have no causal connection to the physical world.
2. It would be surprising to find that what arises from the aesthetic impulse in humans and has no causal connection to the physical world should be significantly effective in physics.
3. Therefore, it would be surprising to find that mathematical concepts should be significantly effective in physics.
4. The laws of nature can be formulated as mathematical descriptions (concepts) which are often significantly effective in physics.
5. Therefore, it is surprising that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.
6. Therefore, the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics merits explanation.

7. Theism provides a better explanation of the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics than does atheism.

8. Therefore, the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics provides evidence for theism.

It would be helpful to have a simpler formulation of this argument. It seems to me that we would not be misleading to epitomize our argument as follows:

1. If God does not exist, the applicability of mathematics is just a “happy coincidence.”

2. The applicability of mathematics is not just a “happy coincidence.”

3. Therefore God exists.

The idea is that on naturalism, whether realist or anti-realist, there is no explanation of mathematics’ applicability and therefore it is just a happy accident. But given the accuracy and elegance of the mathematics involved in the physical laws of nature, such a position is explanatorily inadequate and implausible. That the universe was therefore created by a transcendent, personal being of enormous power and intelligence who made the universe to operate according to the laws he had in mind is the best explanation. To recall the words of Paul Dirac, “God is a mathematician.”<sup>17</sup>

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<sup>1</sup> Eugene Wigner, “The Unreasonable Effectiveness of Mathematics in the Natural Sciences,” in *Communications in Pure and Applied Mathematics* 13/1 (New York: John Wiley & Sons, 1960).

<sup>2</sup> Wigner, “Unreasonable Effectiveness of Mathematics,” p. 2. Wigner re-phrases this first point as “mathematics plays an unreasonably important role in physics.”

<sup>3</sup> *Ibid.*, p. 8.

<sup>4</sup> *Ibid.*, pp. 2-3.

<sup>5</sup> *Ibid.*, p. 131.

<sup>6</sup> Penelope Maddy, “Believing the Axioms I,” *Journal of Symbolic Logic* 53/2 (1988): 481-511.

<sup>7</sup> Wigner, “Unreasonable Effectiveness of Mathematics,” p. 3.

<sup>8</sup> *Ibid.*

<sup>9</sup> Even those who maintain that mathematics plays an explanatory role in science (e.g., Manfred R. Schroeder, “The Unreasonable Effectiveness of Number Theory in Physics, Communication and Music,” *Proceedings of Symposia in Applied Mathematics* 46 [1992]: 1-19, [http://dx.doi.org/10.1090/psapm/046/1\\_195839](http://dx.doi.org/10.1090/psapm/046/1_195839); Alan Baker, “Are there Genuine Mathematical Explanations of Physical Phenomena?” *Mind* 114/454 [2005]: 223-238; Marc Lange, “What Makes a Scientific Explanation Distinctively Mathematical?,” *British Journal for the Philosophy of Science* 64/3 [2013]: 485-511) recognize that such is a case of acausal explanation. It amounts to nothing more than the broadly logical necessity of mathematical truths, e.g., Mother cannot divide 23 strawberries among her 3 children evenly because 23 is not divisible by 3 without remainder. Lange writes, “these explanations explain not by describing the world's causal structure, but roughly by revealing that the explanandum is more necessary than ordinary causal laws are. . . . These necessities are stronger than causal necessity, setting distinctively mathematical explanations apart from ordinary scientific explanations” (Lange, “What Makes a Scientific Explanation Distinctively Mathematical?,” p. 491).

<sup>10</sup> Wigner, “Unreasonable Effectiveness of Mathematics,” p. 7.

<sup>11</sup> Wigner, “Unreasonable Effectiveness of Mathematics,” p. 14.

<sup>12</sup> *Ibid.*, p. 2.

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<sup>13</sup> Mary Leng, *Mathematics and Reality* (Oxford: Oxford University Press, 2010), p. 239.

<sup>14</sup> Balaguer, *Platonism and Anti-Platonism in Mathematics*, p. 136.

<sup>15</sup> “On the Foundations of Physics,” July 5, 2013,  
<http://www.3ammagazine.com/3am/philosophy-of-physics/>.

<sup>16</sup> The turning point from examples of descriptive applicability which Steiner deems not mysterious because they can be explained in terms of physical properties of nature to examples of descriptive applicability which do seem mysterious because they have no physical basis, occurs at pp. 35-36 of Steiner’s book. For discussion of the following examples, see Steiner, *Applicability of Mathematics*, pp. 36-40, 95-97, 102.