

Mary Leng's Mathematics and Reality

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Mary Leng's new book is a very thorough and closely argued response from a non-realist perspective to W. V. Quine's Indispensability Argument for the existence of abstract objects, specifically sets.

Incredibly—given its enormous influence—, Quine never clearly articulated or defended at any length his argument for the reality of mathematical objects, leaving it to us to reconstruct the argument as best we can. Here is Leng's formulation of Quine's Indispensability Argument (p. 7):

1. We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe. (Naturalism)
2. The confirmation our theories receive extends to all their statements equally. (Confirmational Holism)
3. Statements whose truth would require the existence of mathematical objects are indispensable in formulating our best confirmed scientific theories. (Indispensability)
4. Therefore, we ought to believe that there are mathematical objects. (Mathematical Realism)

Unfortunately, this formulation of the argument suppresses a distinctive Quinean metaontological thesis, namely, Quine's Criterion of Ontological Commitment, by folding it into Indispensability. Since Leng thinks that mathematical statements are indispensable to our best confirmed scientific theories, she does not think to challenge Quine's criterion. The reason this is unfortunate, I think, is that a truly fundamental critique of Quine's argument will proceed, not merely on the ontological level, as Leng's does (p. 1), but on the meta-ontological level, that is to say, it will examine critically Quine's prescription for settling ontological disputes.

The centerpiece of Quine's meta-ontology is his Criterion of Ontological Commitment. Although Quine was not always careful about the formulation of his criterion, a working version would be as follows:

5. We are ontologically committed to the values of variables bound by the quantifiers in a first-order symbolization of a true, canonically formulated scientific statement. (Criterion of Ontological Commitment)

Though widely assumed, Quine's criterion is eminently challengeable. These challenges receive only passing notice in §1.2 "A Note on Existence and Quantification" (pp. 14-17; cf. §2.2.1). Philosophers of mathematics like Charles Chihara and Geoffery Hellman have provided alternative semantics for

mathematical sentences which allows us to affirm their truth without commitment to mathematical objects. Leng simply “sets aside” such proposals because she wants to challenge the claim that our best scientific theories commit us to the *literal* truth of mathematical statements. Here she misstates the issue; for Chihara and Hellman do take mathematical statements to be literally true—they just provide a different semantics. It is, for example, literally true on Hellman’s view that “If there were numbers, then there would be prime numbers greater than 100.” What Leng really means is that she wants to challenge the claim that given the standard semantics for existential quantification and singular terms, we are committed by our best scientific theories to mathematical objects. That just is to proceed on the ontological level and to ignore the more fundamental issues of meta-ontology.

Other meta-ontological challenges to Quine’s criterion deny that the existential quantifier is ontologically committing. That the ordinary language expression “there is/are,” which is codified by the existential quantifier “ \exists ,” is not ontologically committing is indisputable. We truly say, for example, “There is an easier way to solve this equation” without thereby committing ourselves to reality of objects called “ways.” So why think that we are committed to the existence of the values of the variables bound by the existential quantifier in true theoretical sentences? Leng recognizes that some philosophers want to employ an existence predicate “ E ” in place of the existential quantifier to express ontological commitments. Her gloss on this is:

If we allow for *non-existent objects* in our domain of quantification, or alternatively reject the view of quantifiers as ranging over a domain of objects, the truth of claims such as M might not require the *existence* of any mathematical objects (despite asserting that *there are* such things) (p. 15).

The first alternative is neo-Meinongianism, according to which “ \exists ” is ontologically neutral and abstract singular terms do refer to objects, albeit non-existing objects. The second alternative is substitutional quantification, which does not take bound variables to range over a domain of objects, but takes them to be placeholders for linguistic expressions. Leng says nothing further about substitutional quantification, which philosophers like Dale Gottlieb have gainfully employed to avoid ontological commitment to mathematical objects. She associates a third alternative, Jody Azzouni’s Neutralism, with the two afore-mentioned approaches, despite the fact that Azzouni explicitly rejects non-existent objects and maintains objectual quantification to be ontologically non-committing. Leng misinterprets Azzouni to claim that “there is” is not univocal in its meaning (p. 16), thereby conflating his Neutralism with yet another alternative, Quantifier Variance, defended by Eli Hirsch, which holds that such existential locutions are equivocal. Azzouni is quite clear that Neutralism makes no such claim but simply holds that both the ordinary language “there is/are” and “ \exists ” are ontologically neutral. In the end Leng chooses to ignore these fundamental, meta-ontological challenges to Quine’s criterion because such responses to the Indispensability Argument “would be liable to obscure rather than illuminate matters” (p. 17). I could not disagree more. While these responses raise very difficult questions, they

shed light on the meta-ontological presuppositions of Quine's argument, assumptions which remain safely in the shadows on Leng's approach, where they exert, unchallenged, their surreptitious control over the course of the argument.

For if one eschews any critique of Quine's Criterion of Ontological Commitment one has little choice but to embrace Fictionalism, the view that sentences quantifying over mathematical objects or containing singular terms having as referents mathematical objects are uniformly false. But one has to swallow hard before affirming that it is false that "2+2=4." Such a statement seems not only true, but, given the meaning of the terms, necessarily true. The only reason the Fictionalist denies its truth is that its singular terms "2+2" and "4" must, on the customary semantics, refer to objects if the sentence is true, and the Fictionalist wants, understandably, to deny that there is an object designated by "2+2" or "4". But surely, that ought to lead us to call into question the presupposition of the customary semantics. Indeed, we have a quick and easy argument to that effect:

6. If the customary semantics is correct, then $2+2 \neq 4$.

7. $2 + 2 = 4$.

8. Therefore the customary semantics is not correct.

In an interesting discussion of "The Obviousness of Elementary Mathematics" (§4.3.1), Leng implausibly suggests either that what we find obvious is not the truths of elementary arithmetic, but that they follow from the Peano axioms or else that it is due to our childhood conditioning that we find it hard not to believe in numbers. Nonsense; most people who believe $2 + 2 = 4$ have never heard of the Peano axioms and have never thought of numbers as objects. She suggests more plausibly that we find arithmetic obvious because of counting, and this involves only adjectival use of numerical terms, which is not ontologically committing. Right; and that goes to reinforce my point that we do find elementary arithmetic obviously true and do not think it to involve the ontological commitments which the customary semantics would foist upon us. We have, then, powerful reason to explore the meta-ontological presuppositions of Quine's argument.

Leng's strategy, however, is different. She instead challenges Confirmational Holism, while conceding or even agreeing to Naturalism and Indispensability. Confirmational Holism is crucial to Quine's Indispensability Argument, for without it it is open to the Fictionalist to maintain that even though our best scientific theories may be overwhelmingly confirmed by the empirical evidence, nevertheless the mathematical statements in those theories are false. Confirmation extends only to the nominalistic content of our theories. Therefore, the empirical confirmation of our best theories does not oblige us to believe the mathematical statements of those theories.

Now if the rejection of Confirmational Holism were the sole project of Leng's book, then her treatment might appear to be of little interest for the contemporary debate. For Confirmational Holism is widely recognized to be the most implausible of Quine's theses and finds almost no defenders today. As Elliott Sober points out, even if we acquiesce to a non-distributive confirmational holism, whereby theories are confirmed or disconfirmed as wholes, that does nothing to sanction distributive confirmational holism, the claim that the confirmation of a theory as a whole is distributed equally to each of its constituent statements, a doctrine with untenable consequences. So Quine's original Indispensability Argument is now obsolete. Contemporary indispensability arguments for mathematical realism therefore place no reliance on Confirmational Holism. Fortunately, Leng's wide-ranging discussion ventures far beyond a mere repudiation of Confirmational Holism and speaks to issues at the heart of current debate.

Although Leng characterizes her view as Fictionalism, such an appellation is, I think, misleading. Standard Fictionalism, as expounded, for example, by Mark Balaguer, holds that mathematical sentences quantifying over mathematical objects or containing singular mathematical terms are false. Moreover, he emphasizes that Fictionalism has little to do with fiction as a literary genre. By contrast, Leng's position is that our best scientific theories give us no reason to believe that mathematical sentences are true. Her position, at least until the fifth to the last page of the book is merely agnostic. Moreover, her view appropriates significantly Kenneth Walton's analysis of fiction as involving make-believe, holding that we should regard statements of applied mathematics as examples of prop-oriented make-believe. Since literary theorists recognize that it is not essential to fiction that the discourse be false, Leng contends that correctly regarding mathematical statements as fiction does not imply their falsity. The mathematical statements in our best scientific theories could, then, be true, as the realist claims, but we do not have any reason to think that they are (p. 207). Leng's view is, thus, not standard Fictionalism but a version of what one might call Pretense Theory. Mathematical statements in science are best regarded as asserted under the pretense that there are mathematical objects and so may be said to be fictionally true (or, as she puts it, fictional).

Leng softens the ground for her Pretense Theory of mathematical statements through an interesting discussion of the role of idealizations in scientific theories. She shows that scientists are, to borrow Quine's phrase, "up to their necks" in idealizations, such as ideal fluids or gases, which falsely model real fluids and gases as continuous. Quine's bet (which he never tried to make good on) that all such idealizations could be eliminated in a canonical language of science Leng deems "vastly overoptimistic" (p. 117). Quite the contrary, she contends, idealization is indispensable to successful science. But if that is the case, then Confirmational Holism fails, for a highly confirmed scientific theory may contain statements which are literally false. In the case of idealizations, scientists make believe, for example, that fluids are continuous and study them under that pretense, in virtue of which they can make dependable predictions about the behavior of real world fluids.

So which of our entities quantified over in theoretical statements should we regard as real and which as fictional? Leng advises that we examine our theoretical posits to see if they are postulated merely for practical advantage. Could we make use of a theory even if there were in fact no such objects? If we cannot, that gives good reason for taking such postulates literally. For that reason Leng rejects Bas van Fraassen's constructive empiricism, for there must be something like electrons to explain phenomena like tracks in a cloud chamber. By contrast, Leng argues, mathematical objects fail to meet this criterion. Mathematical objects in scientific theorizing serve merely as representational aids which would be useful even if there were no such objects. They can therefore be regarded as fictions.

Leng thinks that her Pretense Theory prescribes how we *should* understand science. The Pretense Theorist will not *believe* our scientific theories, but he will *accept* them, where accepting a theory means believing its nominalistic content (p. 200). In response to the challenge that to accept a theory just is to believe it, Leng attempts to turn the tables by arguing that allegedly realist scientists do not really believe their theories, as is evident in the various ways their mathematical statements are made and regarded. Her argument strikes me as highly implausible, treating scientists as deluded or insincere when making assertions like, "The temperature of this liquid is 48 degrees." Far more probable, I think, is that, once again, scientists do not believe the customary semantics that mandates that the truth of such mathematical sentences requires the existence of mathematical objects.

Leng devotes a chapter to explaining the success of mathematics. Realists can account for the predictive success of science because, on their view, our scientific theories are true, whereas the predictive success of science remains a mysterious coincidence on constructive empiricism. Leng argues that the Pretense Theorist can match the realist in accounting for science's success, since the truth of the mathematical statements of a theory have no effect on the non-mathematical statements. So long as the causally relevant nominalistic content of science is accurate, the relevant theories will be successful.

It will be recalled that Pretense Theory does not imply that the mathematical statements featured in our scientific theories are false: they are fictionally true, which is compatible with their being literally true. But as her *coup de grâce*, Leng offers a sort of Dispensability Argument against platonism. Since we are able to get along quite nicely without mathematical objects, Ockham's Razor enjoins us to dispense with them. Hence, we should be not merely agnostic but anti-realist about the existence of mathematical objects. This argument seems to assume that mathematical objects, if they existed, should make more of a difference to scientific theorizing than they do—but why think that?

Moreover, one might think that such a dismissal of abstract objects is much too quick, since it ignores any potential extra-scientific grounds for embracing abstract objects. But that brings us to a fundamental feature of Leng's treatment on which I have not commented, namely, her Naturalism. Her

investigation of the topic at hand is carried out entirely within the parameters of Quine's naturalized epistemology. This has both advantages and disadvantages for Leng's project. On the plus side, it gives Leng a place at the table reserved for the in-crowd of naturalistic philosophers, rather than leaving her excluded as one coming from the standpoint of what has been called "an alienated epistemology." On the negative side, it hamstrings her in the search for the truth about the questions she explores. Naturalized epistemology treats the natural sciences, broadly construed, as the only basic source of knowledge. So constricted a methodology peremptorily closes off inquiry. For example, if natural science finds no place for the reality of tense and temporal becoming, as Quine believed, then we shall be forced to regard these as illusory. Ethical values and moral duties, playing no role in natural science, will similarly be dismissed. Interesting metaphysical questions like "Do composite objects exist?" will be short-circuited by scientific theories quantifying over such objects. Theological questions about God's relationship to abstract objects will become undiscussable.

Why adopt so constrictive a methodology as Naturalism? Although Leng devotes a chapter to this question, dismissing along the way a "bad argument" based on the success of science and the failure of philosophy, she never successfully addresses the question of why we should limit our basic sources of knowledge to the natural sciences. Why not accept other sources as well? At one point she poses the question, intriguing in light of Alvin Plantinga's Reformed Epistemology, "Why, then, look to 'science', however broadly construed, rather than, for example, the framework provided by a particular theological worldview?" (p. 35). Plantinga, of course, rejects the classical foundationalism which Quine abandoned in taking natural science to be a basic source of knowledge, but he does not, like Quine, limit our basic sources of knowledge to the deliverances of natural science. Leng answers her own question:

Quine's reason for looking to science, in particular, to discover what we ought to believe that there is is just that it is our current best science that is the result of our most concerted efforts at refining and improving our conceptual scheme in describing and systematizing our experience. If, as naturalized philosophers, we take our cue from our scientific theories and methods, rather than seeking to abandon them, we can hope to contribute to this internal refinement of our current state of reasonable belief rather than seek to undermine it (as, for example, not *really* reasonable (p. 35).

The false dichotomy of the second sentence above is glaring. One who accepts additional sources of knowledge, for example, rational intuition, as basic in no way seeks to abandon scientific theories and methods or to undermine our current state of reasonable belief! And the first sentence is plausibly over-restrictive in its description of human experience, ignoring as it does our experience of tense, moral values, the self, and so on.

When Leng concludes on the chapter's final page, "I have presented in this chapter a defense and clarification of P1 (Ontological Naturalism)" (p. 44), my reaction was surprise because no such defense

had been proffered. Leng fails to address the fact, stressed by Michael Rea in his *World without Design*, that the only plausible construal of Naturalism is that it is a methodological disposition to accept natural science alone as a basic source of knowledge and is, as such, incapable of justification. The decision to pursue the ontological question of the reality of mathematical objects using only the deliverances of natural science is arbitrary.

As a non-Naturalist, I think we have good theological grounds for denying the existence of abstract objects such as mathematical objects, namely, their incompatibility with God's aseity and divine *creatio ex nihilo*. My interest, then, is to find an anti-realist view that best comports with those doctrines as well as with the deliverances of science and rational intuition. Moreover, I think that the theist enjoys a considerable advantage over the Naturalist in explaining the uncanny success of mathematics. As Leng points out, for the non-theistic realist, the fact that physical reality behaves in line with the dictates of acausal mathematical entities is "a happy coincidence" (p. 239). But the theistic realist can argue that God has fashioned the world on the structure of the mathematical objects. Leng says that on Fictionalism mathematical relations just mirror the relations obtaining among things in the world, so that there is no happy coincidence. Well and good, but what remains wanting on secular Fictionalism is an explanation why the physical world exhibits so complex and stunning a mathematical structure in the first place. The theistic Fictionalist, on the other hand, can maintain that God has constructed the world on the fictional blueprint conceived by Him.

Now I appreciate that Leng enjoys a dialectical advantage by staking out epistemological common ground with her secular interlocutors. But truth should not be sacrificed for the sake of expediency. Besides, one need not embrace Naturalism in order to point out the weaknesses of Confirmational Holism or to argue that there are no good scientific grounds for positing the existence of mathematical objects.