Review: An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science of Quantity and Structure

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SUMMARY

Review: *An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science of Quantity and Structure* by James Franklin.

James Franklin aspires to a realist view of mathematical objects as concrete, rather than abstract, objects. It is shown that he fails to carry out his program but is forced to revert to Platonism.

REVIEW: AN ARISTOTELIAN REALIST PHILOSOPHY OF MATHEMATICS: MATHEMATICS AS THE SCIENCE OF QUANTITY AND STRUCTURE

My interest as a philosopher of religion in Franklin's book arises out of my quest for a plausible alternative to a Platonist account of mathematical objects, one that, unlike Platonism, will be compatible with God's being the sole ultimate reality. Although Franklin does not address theological questions, his Aristotelian realism might seem at first blush to offer one such alternative. For he rejects the existence of abstract objects, maintaining that mathematics is about the concrete world. He reductively analyzes mathematical objects in terms of concrete properties and relations. Since the concrete world is created by God, the theist philosopher might hope that Franklin's anti-Platonist realism would be consistent with theism's doctrine of divine aseity and *creatio ex nihilo*. Whether such a view can avoid the well-known bootstrapping objection facing absolute creationism remains a moot question.

Contemporary realists tend to be Platonists about mathematical objects. What few anti-Platonist realists there are tend to be divine conceptualists, who hold that mathematical objects are thoughts in God's mind. But the idea that mathematics is about physical objects finds few contemporary proponents. Franklin acknowledges that such a viewpoint is all but invisible in contemporary philosophy of mathematics (p. 105). Indeed, Franklin's own view becomes so qualified in the course of its exposition that it is questionable whether it can avoid collapse into Platonism or anti-realism (nominalism).

Franklin is a professional mathematician, not a philosopher. He upbraids contemporary philosophers of mathematics for failing to keep up with developments in mathematics and the formal sciences.

> The traditional diet – numbers, sets, infinite cardinals, axioms, theorems of formal logic – is far from typical of what mathematicians do. It has led to intellectual anorexia, by depriving the philosophy of mathematics of the nourishment it could and should receive from the expansive world of mathematics of the last hundred years. Philosophers have almost completely ignored not only the broad range of pure and applied mathematics and statistics, but a whole suite of 'formal' or 'mathematical' sciences that have appeared only in the last eighty years. . . .
>
> . . . It is a pity that philosophers have taken so little notice of them, since they provide exceptional opportunities for the exercise of the arts peculiar to philosophy (p. 82).

We can be grateful for Franklin's admonition, and, indeed, his illustrative use of these various sciences such as control theory, game theory, and computer science is a fascinating feature of his book.

At the same time, philosophers might justifiably complain that Franklin fails to exhibit the philosophical care that analytic philosophers are used to when he treats philosophical positions and problems. For example, Franklin does not carefully distinguish his view from Platonism and from various anti-realisms (which he tends to lump indiscriminately together under the label "nominalism"). He often contrasts Aristotelianism with Platonism by saying that according to Aristotelian realism mathematical and all other properties can be “instantiated” or “realized” in physical (or any other) reality (pp. 2-3). But don’t Platonists also hold that there are property instances (like this redness of my rose), so that properties can be said to be instantiated or realized in physical reality? On Platonism abstract properties are instantiated in concrete things by being exemplified by those things. Franklin is using words like “instantiated” and “realized” in an idiosyncratic sense to indicate that universals are concrete, even physical, realities.

Franklin also fails to differentiate the senses in which the term “nominalism” is used. With respect to the medieval debate over universals, someone who denies the reality of universals is a nominalist, even if he accepts the reality of abstract objects such as classes. But in the contemporary debate over the existence of mathematical objects, a person who believes in concrete universals usually counts as a nominalist, so long as he rejects the reality of abstract objects. Franklin conflates the contemporary and medieval debates when he says,

> If Platonism was taken to mean ‘there are abstract objects’ and nominalism to mean ‘There aren’t’, then it can appear that Platonism and nominalism are mutually exclusive and exhaustive positions. However, the words ‘abstract’ and ‘object’ both work to distract attention from the Aristotelian alternative: ‘abstract’ by suggesting a Platonist disconnection from the physical world and ‘object’ by suggesting the particularity and perhaps simplicity of a billiard ball (p. 14).

Clearly, Platonism and nominalism as defined in the first sentence are mutually exclusive and exhaustive positions. In this sense Franklin’s concrete realism is a form of nominalism, even if he holds to the reality of immanent universals. Moreover, concrete universals are, in contemporary parlance, objects (entities), even if they are not particulars. As Franklin himself asserts, Aristotelianism replaces abstract objects with “mind-independent objects which are spatiotemporal and causal, namely relations such as ratios” (p. 240).

Franklin's view, then, is that universals are concrete objects immanent in things. One would expect him therefore to have something to say about the problem of how a concrete universal can be multiply instantiated, that is to say, exist wholly at distinct places in space. The Platonist faces no such conundrum, since his abstract universals have multiple, distinct instances in the physical world. But some explanation is in order for how any concrete object can exist wholly at separated places. Unfortunately, Franklin does not even address this question, apart from a passing endorsement of David Armstrong's view "that the basic structure of the world is 'states of affairs' of a particular's having a universal" (p. 12). Armstrong himself, however, admits that he cannot explain how concrete universals can be multiply instantiated.

The proponent of concrete universals must also confront the problem of uninstantiated universals. This problem is especially acute for a concretist account of mathematics, since the finite world cannot accommodate the infinities of classical mathematics. Franklin abandons a "strict this-worldly Aristotelianism, according to which uninstantiated universals do not exist in any way" in favor of a "semi-Platonist or modal Aristotelianism. . . , according to which universals can exist and be perceived to exist in this world and often do, but it is a contingent matter which do so exist, and we can have knowledge even of those that are uninstantiated, and of their necessary interrelations" (p. 26). Such a view is said to contrast with "(extreme) Platonism, according to which universals are of their nature 'abstract objects', that is, they are not the kind of entities that could exist (fully or exactly) in this world, and they lack causal power" (Ibid.). At first blush Franklin's view appears to be the extraordinary doctrine that uninstantiated universals are only contingently abstract, that is to say, they exist and can be known, but they can turn into concrete universals. The Platonist errs in thinking universals to be essentially abstract and causally effete; rather they can become concrete (instantiated), in which case they become sense perceptible and causally efficacious. Such a bizarre view has the implication that when certain things (say, dodos) cease to exist, then certain concrete universals revert to being once more abstract. While such a view would not be your typical Platonism, it hardly deserves to be called Aristotelianism or be classified as a form of concrete realism.

Fortunately, such is not Franklin's meaning. For he seems to be diffident, after all, about the reality of uninstantiated universals. He asks,

> Should an uninstantiated universal be said to 'exist'? That is not regarded as a meaningful question by the semi-Platonist Aristotelian. When a universal is instantiated in a particular in some state of affairs, a being exists with that universal; when a universal is not instantiated, there are knowable possibilities concerning it and its relation to other universals, but there is no need to grant it an 'existence' parallel to that of particulars. It may be convenient to set up names and mathematical notations for such possibilities, but it is not the business of the philosophy of universals or the philosophy of mathematics to deal with complex questions in the philosophy of language concerning reference to objects beyond the here and now (such as fictional and future objects, as well as possibilities) (pp. 28-9; *cf.* p. 239).

This is an astonishing paragraph. A great deal of contemporary philosophy of mathematics and of universals deals with complex questions in the philosophy of language concerning reference to objects beyond the here and now. Such questions are inescapable for any would-be adequate philosophy of mathematics. In suggesting that the question of the existence of uninstantiated universals is meaningless, Franklin appears to endorse arealism, the view that there is no fact the matter with respect to the existence of abstract objects. But, as becomes evident in the sequel, that is not his meaning. Franklin himself later provides an anti-realist account of zero and the empty set as merely useful fictions (pp. 234-9). Indeed, his remarks here about knowable possibilities sound very much like Geoffrey Hellman's modal structuralism, which is a sort of counterfactual If-thenism concerning mathematical entities.

It is therefore intriguing that later in the book Franklin acknowledges that Hellman's modal structuralist theory "is the closest to that of the present book" (p. 117). But he voices three objections to Hellman's view: (1) Hellman's "excessively hypothetical" interpretation of arithmetic sentences "is correct of uninstantiated structures, but avoids mention of what happens when the structures are in fact instantiated" (p. 118). (2) Hellman's theory involves "a hidden reference to realistically interpreted universals," for his universal quantifiers range over classes (Ibid.). (3) Like logicism, Hellman's project runs afoul of the non-logical nature of the Axiom of Infinity, for Hellman postulates the logical possibility of an infinitude of atoms, "but it is implausible that this possibility is in any sense a matter of logic" (p. 119).

These objections seem wholly misconceived. (1) If the antecedents in Hellman's counterfactual conditionals are true, then of course they are informative of actual structures (*cf*. Franklin's own remarks on p. 76 on universal conditionals' being about what is real). (2) Second order universal quantification over classes is on no account ontologically committing, as Franklin himself acknowledges (p. 235). (3) Postulating the mere possibility of an infinitude of objects, in contrast to the non-modal Axiom of Infinity, is a matter of either strict or broad logical possibility.

Now if Hellman's modal structuralism gives an adequate account of so-called uninstantiated universals which is mathematically adequate, then the question arises as to why we should be realists at all. Why include concrete universals in our ontological inventory? It is noteworthy that Franklin does not embrace the most popular rationale for realism about mathematical objects, namely, the Indispensability Argument. He denies that first-order quantifiers and singular terms are devices of ontological commitment. "Ontology is not subject to the vagaries of language in that way" (p. 115). Citing Jody Azzouni, Franklin says, "It may be that the way language works requires names for or quantification over 'beings' that the users of the language know well are not real" (p. 235).

So why be a realist? Remarkably, Franklin has almost nothing to say in response to this question. All I could find was a claim that anti-realism (nominalism) could not solve the One over Many problem: "The main problem for nominalism is its failure to give an account of *why* different individuals should be collected under the same name (or concept or class), if universals are not admitted" (pp. 12-13). But Franklin gives no argument that two things' being white, for example, requires that there be literally some other thing which is identical in the two things.

Franklin claims that mathematical properties and relations are sense perceptible, since they are physical. He says that "perception of the simpler quantitative properties of physical things is as direct and straightforward as perception of color and hardness" (p. 176). Certainly, we perceive that there are, say, two dogs just as we perceive that the dogs are brown. But neither perceptual truth requires commitment to the reality of properties. One could even say that we perceive that the number of the dogs is two or that the color of the dogs is brown, but, absent the disputed criterion of ontological commitment at play in the Indispensability Argument, such singular terms are no more ontologically committing than the adjectival terms.

Franklin also defends the claim of the early Penelope Maddy that sets are sense perceptible (pp. 174-5). Such an outlandish claim fails to reckon with the strange properties of sets. For example, sets have their members essentially (Axiom of Extensionality). Even if we perceive aggregates of things, we do not perceive that those aggregates have their members essentially and are therefore sets. Franklin asserts that "The relation of a platoon to a brigade is numerical because they are both sets of soldiers" (p. 39). This is false, since platoons and brigades do not have their members essentially. Later Franklin claims that "The set of blue things is not the property blue nor is it in any sense an analysis of the concept blue. It is the property blue that pre-exists and unifies the set and supports the counterfactual that if anything else were blue it would be a member of the set" (p. 105). This assertion not only violates the Axiom of Extensionality but also seems to presuppose a principle of universal comprehension, according to which properties determine sets, and so leads to the paradoxes of naïve set theory. When Franklin says that we perceive how a heap is "divided by a unit-making property [like *being an apple*], and that is all there is to being a set" (p. 175; *cf*. p. 16), he is using the word "set" in an idiosyncratic sense (*cf*. p. 60). As Maddy herself came to see, we cannot be rightly said to perceive sets.

So there really isn't much of a case made for realism in Franklin's book. One is therefore rather startled to read late in the book the statement, "If the Aristotelian is prepared to admit a fictionalist theory of zero and the empty set, was it really necessary to expend so much effort defending realism and fending off fictionalism up to that point?" (p. 239). What effort? Franklin's remark about fending off fictionalism suggests that he has conflated two senses of realism: alethic realism and ontic realism. I suggest that Franklin's book really amounts to a defense of alethic realism concerning mathematical statements, not a defense of ontic realism about concrete universals. His concern to fend off fictionalism is a concern to defend the truth of mathematical sentences, pure and applied. But absent the criterion of ontological commitment underpinning the Indispensability Argument, there is no reason to agree with the fictionalist that mathematical truths commit us to objects like properties and relations. Franklin summarizes his argument by saying, "What has been asserted is that there are properties, such as symmetry, continuity, divisibility, increase, order, part and whole, which are possessed by real things and are studied directly by mathematics, resulting in necessary propositions about them" (p. 81). Given Franklin's denial that informal quantifiers like "there are" are devices of ontological commitment, this statement could have been made by an anti-realist, so long as he is not a fictionalist.

In one sense, Franklin's book can be seen as an extended discussion of mathematics' applicability to the world. Franklin's answer to Eugene Wigner's famous puzzle of the unreasonable effectiveness of mathematics is that mathematics is a science of physical objects. The world itself is a mathematical object, and therefore mathematical theorizing can apply to it. This answer, apart from the problems concretism in mathematics occasions, only pushes the question back a notch. Why does the world have the structure it does? Franklin argues that in some cases, such as elementary arithmetic, it is impossible for mathematics not to apply to the world. But in most cases, he admits, physical reality does not have to have the mathematical structure that it does. Franklin ventures no further than this. But explaining the mathematical structure of the actual world may lead us beyond naturalism to theism.

Footnotes

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