Review: Natur und Zahl: Die Mathematisierbarkeit der Welt by Bernulf Kanitscheider

William Lane Craig
Used by permission of *Philosophia Mathematica* 24/1 (2016): 136–41

SUMMARY

A critical review of a prominent, non-theistic, German philosopher's attempt to articulate a philosophy of mathematics consistent with naturalism, an attempt which, it is argued, fails, both with respect to the ontology of mathematical objects and the applicability of mathematics to the physical world.

REVIEW: NATUR UND ZAHL: DIE MATHEMATISIERBARKEIT DER WELT BY BERNULF KANITSCHEIDER

The title of Bernulf Kanitscheider's *Natur und Zahl* has an even better ring in English: *Nature and Number*. The book is a wide-ranging discussion of the mathematical nature of the physical world. It takes the form of a sort of perambulation through history, beginning with the ancient Pythagorean philosophers, with stops along the way to note analogies or parallels in contemporary science, especially cosmology and astrophysics, Kanitscheider's area of specialization, in order to illustrate the viewpoints under discussion. Unfortunately, this procedure results in a rambling and unsystematic exploration of the issues and eventually breaks down in the later chapters, where Kanitscheider tends to desert the historical approach in favor of a more thematic discussion.

The book presents itself as an exploration of the reason for the astonishing applicability (*Anwendbarkeit*) of mathematics in physics, a question neglected, according to Kanitscheider, in Germanic philosophy in contrast to Anglophone philosophy (pp. 12-13). In fact, however, Kanitscheider treats this issue only tangentially in the book. The actual question with which he struggles is the compatibility of mathematics with naturalism. Kanitscheider is a committed naturalist, and he is acutely aware of the problems for his metaphysical worldview posed by mathematical truth (which he wants to affirm) and the underlying ontology of mathematical Platonism (which he wants to avoid). Thus, the book's center of gravity is its longest chapter "Naturalism in the World of Mathematics.[1]

It is important to understand that by "naturalism," Kanitscheider does not mean what many contemporary philosophers of mathematics mean by that term: the view that each discipline, in particular mathematics, has the right to proceed and be tested by its own internal standards without imposition from outsiders of extraneous standards. Neither does he mean what Quine meant: a naturalized epistemology which takes the idealized sciences to be the only sources of knowledge. Quine affirmed the compatibility of his naturalized epistemology with "positing sensibilia, possibilia, spirits, a Creator," in the unlikely event that there was indirect explanatory benefit in so doing (*Dialectica* 49 [1995]: 295), and, of course, he did admit sets into his ontology, since he held that such mathematical objects are required by the truth of our scientific theories.

Rather by "naturalism" Kanitscheider means that metaphysical worldview which affirms "the primacy of matter in the order of nature" (p. 131), not necessarily in a reductive sense (p. 119), but in the sense that "matter-energy represents the supporting substratum of the world's processes and that this fundamental substance is governed by unchanging conservation laws" (p. 120). Kanitscheider himself does seem to espouse a reductive materialism, for he affirms that mental objects are neural processes and that a mathematical fiction would have to exist in the brain as a concrete neural object (pp. 267-8). He goes so far as to say that the "I" (*Ich*) does not exist and so speaks consistently, not of how I might have access to mathematical objects but of how the brain could have access to them or be impacted by them (pp. 111-12).

Kanitscheider takes Platonism to be "the very antithesis of naturalism," since it can hardly be doubted that "the claim of the primacy of matter in the order of nature is irreconcilable with the primordial, isolated existence of conceptual objects" (p. 146). He thus seems to find himself at odds with naturalized epistemology, since his materialism represents a form of first philosophy which is not read off of the sciences but is in fact challenged by them, as he himself recognizes, due to the indispensability (*Unvermeidlichkeit*) of mathematical terms in physical theories and so must be imposed on the sciences in order to preclude, not merely immaterial selves and spirits, but independent mathematical objects. Thus, he insists that "a satisfactory philosophy of mathematics should in any case be compatible with naturalism and one can hardly imagine this other than under the prerequisite of a monistic ontology" (p. 377). The burden of Kanitscheider's book is to find some such philosophy of mathematics.

Unfortunately, he does not explore in any depth various alternatives to Platonism which the anti-Platonist might adopt, and his brief comments on these are scattered throughout the book. He dismisses the medieval view that mathematical objects are grounded in the divine mind as "no longer acceptable in today's secular world" (pp. 92-3) and therefore "obsolete," as well as incompatible with materialistic naturalism (pp. 204-5). More substantively, he objects that God would be as causally isolated from the world as Platonic objects and therefore just as unknowable (p. 211). But this objection fails to reckon with the medievals' conviction that God, as a concrete object, is invested with causal powers in a way that an abstract object is not and has revealed Himself in the world.

Fictionalism might seem more congenial to the naturalist. But Kanitscheider thinks that the value of “a unified semantics for all of the sciences” cuts against “that nominalistic fictionalism, according to which all existential mathematical sentences (such as ‘317 is a prime number’) are fundamentally false because there are no corresponding objects or because they can be regarded as true only in the fairytale land of mathematics” (p. 203). This complaint seems to be misplaced, since one of the advantages fictionalists claim for their view is precisely that they have the same objectual semantics of quantified sentences as the Platonist, disagreeing only as to the truth value of such sentences.

In a later chapter, Kanitscheider complains that fictionalism can distinguish between falsehoods like 5+7=12 and falsehoods like 5+7=13 only by a contextual reference to current mathematics, which is justified merely pragmatically (p. 267). What, then, is the problem with differentiating falsehoods as holding or not holding in the standard model of arithmetic? Kanitscheider says that if all abstract objects are of the same character as Little Red Riding Hood, then they would be useless for the natural sciences. “They must be of a certain sort and, in particular, of just the sort that makes current mathematical theories true” (p. 267). But the fictionalist denies that current mathematical theories are true, just useful; and it is the way the physical world is that makes some mathematical theories more useful than others.

As the above reference to Little Red Riding Hood indicates, Kanitscheider tends to conflate fictionalism with what has been called fictionism or pretense theory, which treats abstract objects on the analogy of fictional characters. Here Kanitscheider expresses reservations about the ontological status of fictions, since they lack identity conditions (p. 266). But pretense theorists like Kendall Walton should not be understood to ascribe any sort of ontological status to fictions. Rather what is fictionally true is what is prescribed to be imagined to be true. We can imagine, for example, that the universe of sets determined by the axioms of ZFC exists by imagining that the axioms are true, and then we may explore what follows from them. Kanitscheider seems to be guilty once again of reifying fictions when he says that the fictitious abstract objects must be thought and therefore exist as neuronal patterns in the brain, which belies their abstract character (pp. 267-8). No, on a pretense theoretical view fictions are not existing things at all. Kanitscheider is conflating pretense theory with yet another anti-Platonism, namely, psychologism, which takes mathematical objects to be mental objects. Such a view is ruled out, among other things, by Kanitscheider’s reductive materialism (p. 269).

Kanitscheider makes only brief mention of other anti-Platonist views which, like pretense theory but in contrast to fictionalism, are consistent with the truth of mathematical sentences. Charles Chihara’s constructibilism is dismissed on the authority of certain structuralists who object to the introduction of modal objects (p. 265)—as though Chihara took possible worlds to be objects rather than a heuristic device! Stephen Yablo's figuralism, according to which abstract talk is broadly metaphorical, gets a mention but sparks a complaint about the difficulties attending the meaning and truth conditions of metaphorical discourse (p. 229) before being left behind. Significantly, I could find no mention of anti-Platonist views which challenge the neo-Quinean criterion of ontological commitment, according to which singular terms and first-order quantifiers are devices of ontological commitment. Rejecting that criterion would enable Kanitscheider to affirm mathematical truths while denying the supposed ontological commitments of such sentences.

So what view does Kanitscheider adopt? That is frustratingly difficult to say. At several places in the book he expresses sympathy for what he characterizes as Aristotle's position on abstract objects (pp. vii, 94, 146, 174, 221, 375). It is therefore all the more remarkable that in his historical survey Kanitscheider skips silently over Aristotle, providing no exposition of his views at all. Kanitscheider appears to be a realist about mathematical objects, since he wants them as truthmakers of mathematical sentences and takes them to be among the ontological commitments of mathematical sentences he accepts as true. At the same time he makes it quite clear that he rejects a Platonic realm of independently existing abstract objects. So mathematical objects must exist only immanently in the physical world. But that raises the question: are they abstract objects or are they physical objects?

Kanitscheider sometimes speaks as though immanent mathematical objects are, indeed, *abstracta*, their immanence serving to alleviate the problems attending transcendent *abstracta*. He asks,

> But then who compels us to think of the algorithmic structure of the world in terms of the interaction of two realms, one a heavenly world of concepts and relations while the other an earthly world of stuff? What we always grasp in experience and theory is just a unity of both components, a finished hylomorphism, and never a process, a causal interplay of two independent components. Once we give up this heaven-earth antagonism and conceive structured nature as an ontologically unified thing, we no longer run into these causal pathologies (pp. 376-7).

This appears to be an explicit endorsement of taking material things to have an ontological structure of abstract form and matter. He sometimes calls his view "naturalized Platonism" or "matter-immanent Platonism" (p. 146), which might be taken to envision immanently existing *abstracta*. On such a view abstract objects exist spatiotemporally, if acausally.

But wholly apart from the question of how abstract objects can be literally in spatio-temporal things, it seems impossible, given Kanitscheider's brand of naturalism, that mathematical objects be immanent abstract objects. For he insists that all that exists is material objects. Moreover, whether immanent or transcendent, it seems impossible for abstract objects to structure the world or to explain why it is as it is, since abstract entities are, as Kanitscheider recognizes, causally effete and so have no effect on anything. But that is precisely the role Kanitscheider wants immanent mathematical objects to play.

So are we to understand that mathematical objects are, indeed, concrete, material objects? Kanitscheider seems in places to endorse David Armstrong and James Franklin's views that universals and mathematical objects are physical entities of some sort (pp. 203, 215, 221). But then he owes us some explanation of just which material objects mathematical objects are supposed to be. How can pure sets, for example, be material objects? Moreover, we need an explanation of how concrete objects can be multiply exemplified. The whole point of abstract universals was that they can exist wholly in different, non-overlapping spatial locations. But a concrete material object does not seem capable of such a thing. Moreover, the objects of the higher reaches of set theory cannot be identified with any physical objects, as Kanitscheider recognizes.

Whichever view Kanitscheider means to espouse, the higher realms of set theory pose an enormous challenge to any sort of immanentism. For if, as Kanitscheider insists, mathematical entities must have "a derivative, ontologically subordinate status" and do not exist without "material substrates (*Träger*)" (pp. 146-7), then the objects of higher set theory, such as Mahlo cardinals and measurable cardinals, since they lack physical *Träger*, simply do not exist. Some sort of anti-realist treatment of such objects thus forces itself upon the immanentist. But then the question arises: why not apply that same treatment to abstract objects that do happen to have physical *Träger*? Since Kanitscheider rejects any attempt to restrict the universe of sets to V=L, he cannot discriminate among mathematical statements with respect to their truth value based on their having physical substrates or not. So if higher set theory can be true without material *Träger*, why not also lower set theory, for which material *Träger* can be identified?

In one place Kanitscheider characterizes as a "robust realism" the view that "the axioms of set theory no longer describe an intelligible world but rather constitute an explication of the concept of set, and theorems are precisely then true if they follow from sentences employing this concept" (p. 275). This strikes me as an apt description of an anti-realist perspective like postulationalism or pretense theory. Such a view legitimates higher set theory despite its lack of a material substrate but at the same time removes any need for realism about sets at all. Adoption of such a viewpoint to deal with mathematical objects which do not exist immanently cuts the nerve of realism.

Almost lost in all this is the problem that supposedly motivated the book: the applicability of mathematics to the physical world. Kanitscheider thinks that immanent realism is all that is needed to solve the problem. More than once he cites Paul Dirac to the effect that there is a mathematical quality in nature itself (pp. vii, 187, 374). Kanitscheider asserts,

> We reach the best understanding of the outstanding applicability of mathematical formalism through the assumption of an immanent, numerical quality of nature. Instrumentalist interpretations, by contrast, make the success of a theory appear to be a lucky accident, a gift to scientists from a congenial fairy. In any case it remains unsatisfying if we allow the idea of a gift simply to stand (p. 188).

As the allusion to instrumentalism hints, Kanitscheider's main perceived opponent is subjective German idealism, a philosophical current which he believes remains strong to this day. He repeatedly represents the problem of mathematics' applicability to the world as the question why, if the world has no objective structure at all, mathematics is so successful in describing the physical world. In response, he affirms scientific realism and an immanent mathematical structure to the world.

Obviously, this is not how Anglophone philosophers typically understand the problem of the uncanny applicability of mathematics to the physical world. Subjective idealism does not even appear on the radar screen. Rather they want to know why the physical world is structured in such a way that mathematical theorizing can result in successful empirical predictions. Obviously, it does nothing to answer this question to assert that the physical world possesses a mathematical quality or immanent structure. That is precisely the phenomenon crying out for explanation. Indeed, on Kanitscheider's view one is simply left with the gift of a mathematically structured nature with no one to thank for it.

---

[1] All translations mine.