Review of Philosophical Perspectives on Infinity, by Graham Oppy

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SUMMARY

A review of Graham Oppy's impressive survey of concepts of infinity, including its implications for *kalam* arguments for the finitude of the past.

REVIEW OF PHILOSOPHICAL PERSPECTIVES ON INFINITY, BY GRAHAM OPPY

Graham Oppy's book is a wide-ranging discussion of the role played by the infinite in various fields of study, including pure mathematics, physics, probability theory, mereology, and related fields, as well as an examination of various philosophical puzzles posed by the infinite, such as the possibility of so-called super-tasks (performing an infinite number of operations in a finite time).

Wholly apart from its fascinating subject matter, one of the interesting features of this book is its motivation. Oppy informs us that, having published an earlier book criticizing the ontological argument, he turned to writing a similar critique of the cosmological argument, aiming to offer an exhaustive taxonomy of arguments along with a thorough discussion of all the key concepts used in those arguments (p. ix). Realizing that such a task was too much for any individual to handle, he regrouped and decided to choose just one of the key concepts figuring in cosmological arguments and to see how that concept is treated in wider philosophical discussions. He chose the concept of infinity, with the intention of producing a two-volume work entitled *God and Infinity,* the first volume of which was to be the present book. The preliminary volume, however, grew to become an independent work, and it remains to be seen whether the sequel will be written.

Oppy explains that his interest in philosophy of religion stems in part "from the conviction that it must be possible to convince reasonable religious believers that traditional monotheistic *arguments* for the existence of God are worthless" (p. xi). So one of the goals of the projected larger work is to make the case that reasonable religious believers ought to recognize that theistic arguments provide "no reason at all" for reasonable nonbelievers to change their minds. Since one might have thought that reasonable religious believers could differ in their assessment of the worth of natural theology, Oppy's conviction must be the audacious belief that *all* reasonable believers can be convinced of the arguments' worthlessness. Similarly, *any* nonbeliever who changes his mind on the basis of a theistic argument is branded unreasonable. Oppy gratuitously assumes, moreover, that the worth of theistic arguments lies wholly in their evangelistic efficacy vis à vis reasonable nonbelievers, ignoring the benefits of natural theology for believers.

On the other hand, Oppy's interest in philosophy of religion also stems "from the conviction that it must be possible for reasonable nonbelievers to hold that there are reasonable believers." This conviction seems to reveal Oppy's non-evidentialism in religious epistemology. So another goal of the larger project is to show that, despite plausible arguments to the contrary, there are conceptions of the infinite which can be successfully integrated into an orthodox theism. We must await the sequel to cash in that promissory note.

There is an intriguing change of tone as the book progresses. The last line of the book's Preface is the confident assertion: "There is a single concept of the infinite that is required to have application outside the philosophy of religion, and no concept that is inadequate to those external applications can be deemed adequate for the purposes of philosophy of religion" (p. xiii). Early in the book Oppy summarily refutes over a dozen problem cases lodged against the existence of an actual infinite (chap. 3). But as the book unfolds, the tone becomes progressively more circumspect and the conclusions more tentative and uncertain. Indeed, chapter 8 on "Some Philosophical Considerations" leaves one very much in doubt as to whether the actual infinite has even mathematical existence, not to speak of instances in the concrete world. Here are the concluding paragraphs of the book:

Before closing, it is perhaps worth emphasising once more that there seem to be evident costs to every stance that one might take on questions about the infinite.

On the one hand, a blanket ban on the infinite seems to bring crippling difficulties. Infinity is everywhere in classical mathematics. In particular, real analysis provides foundations for everything from the calculus to the mathematical theory of probability. Moreover, infinity is found everywhere in the foundations of science and our ordinary thought about the world: Consider, for example, familiar conceptions of the divisibility of space and time. Because the infinite lurks everywhere both in our ordinary thought about the world and in science, it is very hard to see how we could live without it.

On the other hand, involvement with the infinite brings with it a huge range of difficulties. In particular, there are the many puzzles and paradoxes that have been outlined in the pages of this book. Moreover, there are the many quite fundamental problems that arise for such apparently simple notions as counting, adding, maximising, and so forth. Because we are so firmly wedded to limit notions—"best," "first," "greatest," "maximum," and so forth—that do not sit easily with the infinite, it is very hard to see how we can make our peace with the infinite (pp. 294-5).

Such a conclusion hardly seems to support the view that the religious believer who accepts a cosmological argument for God's existence on the basis of the impossibility of the existence of an

actually infinite number of past events is unreasonable in doing so.

In a sense, then, Oppy's book would be better stood on its head. One would have quite a different impression of the problem cases were we first treated, after the "Mathematical Preliminaries" (chap. 1), to the chapter "Some Philosophical Considerations"—it seems bizarre that not until his penultimate chapter does Oppy introduce such crucial distinctions as that between the potential and the actual infinite—, followed by an inquiry as to whether there are in the real world any *bona fide* instances of the actual infinite (chaps. 4-7), and then finally to a consideration of the puzzles which would ensue were an actual infinite to exist. I think one would come away thinking that some wholly reasonable people reject the existence of the actual infinite. One could not therefore indict proponents of the cosmological argument of irrationality, at least on these grounds.

Those who reject the existence of an actual infinite Oppy classes as either finitists or potential infinitists (pp. 261-4; cf. pp. 244-5). The reader should be alerted to the fact that Oppy's understanding of the potential infinite is not the customary notion of a limit concept, but is construed modally. He applies his taxonomy to both pure and applied mathematics with regard to proponents' views on experience, reasonable belief, and ontology respectively. The use of the term "ontology" in this connection can be misleading. Oppy is not, in fact, exploring the question of which, if any, numbers exist—otherwise, there is no room in his taxonomy for the anti-platonist position that no numbers at all exist, however extensive the universe of mathematical discourse may be. Recognizing that platonism is controversial, Oppy begs off a discussion of it (p. 242); rather his concern is whether putative numbers are really numbers, that is, entities that *would* exist *if* all of classical mathematics were true and the quantifiers used in classical mathematics were ontologically committing (p. 243). What Oppy means by "ontology," then, is really the question of the extent of legitimate mathematical discourse. The anti-platonist need by no means be a finitist or potential infinitist in pure mathematics, since he denies either that the statements of classical mathematics are *literally* true or else that their truth commits one ontologically to the entities quantified over. It is all the more telling, then, that Oppy cannot defeat the finitists and potential infinitists even with respect to the legitimacy of *talk* of the actual infinite. He concludes, "Liberal finitism, potential infinitism, and actual infinitism all have very distinguished contemporary defenders . . . . We are very far from achieving any kind of consensus about infinity in the philosophy of mathematics" (p. 256).

When it comes to infinity in applied mathematics, one's concern is with the question of contingently instantiated infinities (p. 260). Thus the question of the existence of an actual infinite in the realm of mathematical objects like numbers has been passed over—unless one holds with Hartry Field that mathematical objects, if they exist at all, do so contingently. Oppy provides here a similar taxonomy of finitism, potential infinitism, and actual infinitism. Oppy takes the finitist to be

committed ontologically to the theses (i) that the actual world is finite with respect to the number of objects in any domain, with respect to the upper limit to the magnitudes of objects' properties and relations, and with respect to a non-zero lower limit to objects' properties and relations; (ii) that every possible world is also finite in these same respects; and (iii) that there are only finitely many possible worlds. I demur. With respect to (iii) the finitist may hold that there are literally no possible worlds, taking a fictionalist or constructibilist line. Whether he is committed to (ii) will likely depend on whether one identifies metaphysical possibility with the broadly logical possibility employed in possible worlds semantics. As for (i), while the finitist allows only a finite number of objects in the world, I see no reason why the mathematization represented by degreed properties or relations should be regarded by the finitist as a mind-independent feature of reality. He is by no means committed to what Oppy calls "a digital conception of reality" (p. 262), since he may regard wholes as logically prior to the parts into which we mentally (or physically) divide them, rather than as compositions logically posterior to their parts.

An actual infinitist, says Oppy, is committed ontologically to an infinite domain of objects or to an infinite magnitude or to an infinitesimal magnitude. A potential infinitist in the realm of ontology is one who is committed to the truth of claims of the form $\forall\Diamond\exists$, that is to say, claims to the effect that for any natural number there is a possible world in which that number of objects exists, but who denies the truth of any claim of the form $\Diamond\forall\exists$, that is to say, any claim to the effect that there is a possible world in which there are as many objects as all the natural numbers. A major shortcoming of these characterizations, one that will affect many of Oppy's discussions in his book, is that they are *tenseless* and so incapable of handling views of time which regard tense and temporal becoming as objective features of reality and, hence, worlds in which the future is potentially infinite in the sense of growing toward infinity as a limit.

Oppy's failure to deal with the debate over platonism with respect to mathematical objects comes home to roost in his final taxonomy of views of the infinite in the book's conclusion: he lists as the four contending views strict finitism, weak potential infinitism, strong potential infinitism, and (strong) actual infinitism (pp. 291-3). The problem is that these same categories were used to classify positions both with respect to the extent of legitimate mathematical discourse and with respect to the number of contingently existing objects in any domain. Oppy characterizes the strict finitist as having "no proper use of the concept of the infinite;" as rejecting classical mathematics; as allowing only finite domains and magnitudes and only finitely many possible worlds. This conflation of positions with respect to two quite distinct questions leaves no room for someone who is a finitist ontologically but an actual infinitist about mathematical discourse, which he regards as fictional or as not ontologically committing.

Similarly, when Oppy concludes, "If we suppose that we understand classical mathematics, then

either we shall be strong potential infinitists or we shall be strong actual infinitists" (p. 293), he is conflating the debate over what constitutes legitimate mathematical discourse and the debate over ontology. Oppy must be unwittingly supposing that platonism is true and that therefore mathematical discourse is ontologically committing. Why else would he conclude, "To reject the suggestion that it is a contingent matter whether classical conceptions of infinity find application to the extra mathematical world, either we shall be intuitionists or constructivists—hence rejecting classical mathematics and, very likely, classical logic—or we shall be strict finitists" (p. 293)?—in opposition to him who holds that it is metaphysically necessary that no actual infinite exist in the extra-mathematical world, just as it is metaphysically necessary that no mathematical objects exist, despite the quite legitimate use of the fictional or ontologically non-committing language of classical mathematics.

Now if Oppy considers all of these conflated positions be defensible, how much more difficult will it be to refute less radical finitism or potential infinitisms which have made their peace with the language of classical mathematics! Returning to Oppy's final conclusion quoted above, we find that his chief reason proffered on behalf of actual infinitism is that a "blanket ban" on the infinite would cripple classical mathematics. But for the fictionalist or non-committalist, as we have seen, a blanket ban is not in question.

As a second line of defense, Oppy also offers a sort of Quine-Putnam indispensability argument to the effect that our best scientific theories essentially involve infinitary mathematics and therefore anyone committed to the truth of those theories is thereby ontological committed to the actual infinite. It is a shame that Oppy did not discuss this argument at any length in his book. Intriguingly, it does put in a brief appearance on pp. 251-2 in a discussion of how we can be said to reasonably believe the statements of classical mathematics (a central problem, by the way, for a Platonist construal of mathematical objects), and there Oppy *rejects* the argument. His misgivings about the argument, however, have to do with the fact that only a small fragment of classical mathematics is required by our best scientific theories, a fragment which is nonetheless infinitary. So Oppy might think that an indispensability argument does work for the truth of infinitary mathematics, if not the whole of classical mathematics. But then he owes us a much more extended discussion of the argument. The argument presupposes, for example, Quine's Criterion of Ontological Commitment, to the effect that we are ontologically committed to the entities bound by the existential quantifier in the regimented sentences in first-order logic of the scientific theories we regard as true. But Quine's COC has been powerfully challenged on various grounds (see, *e.g.*, Charles S. Chihara, *A Structural Account of Mathematics* [Oxford: Clarendon, 2004] and Jody Azzouni, *Deflating Existential Consequence* [Oxford: Oxford University Press, 2004]). Moreover, the argument presupposes Quine's holism, that confirmation of a scientific theory extends indiscriminately to all

of its statements. It is dubious that the mathematical sentences of a theory are confirmed by a physical theory's success, since they are never disconfirmed by its failure. The fictionalist may accept Quine's COC and yet treat the mathematical parts of a scientific theory as merely useful fictions. Finally, one may avoid ontological commitment by offering an alternative semantics for mathematical statements, as Chihara does, adding a constructability quantifier to the quantifiers of standard first order logic.

It might be said that Oppy is not arguing for the truth of the mathematical statements in physical theories, as in the Quine-Putnam version of the argument, but rather for the reality of the physical entities, *e.g.*, spacetime points, postulated by our true theories. But then it is very difficult to see why we should interpret the posits of such theories realistically. Penelope Maddy provides the following striking illustrations of purely instrumentalist use of infinitary language in physical theory:

If we open any physics text with these questions in mind, the first thing we notice is that many of the applications of mathematics occur in the company of assumptions that we know to be literally false. For example, . . . we assume the ocean to be infinitely deep when we analyze the waves on its surface; we use continuous functions to represent quantities like energy, charge, and angular momentum, which know to be quantized; we take liquids to be continuous substances in fluid dynamics, despite atomic theory. On the face of it, an indispensability argument based on such an application of mathematics in science would be laughable: should we believe in the infinite because it plays an indispensable role in our best scientific account of water waves? (*Naturalism in Mathematics* [Oxford: Clarendon Press, 1997], p. 143).

The notion of a manifold of spacetime points is another example of mathematical modeling that we need not, indeed, should not, take literally, given the metaphysical commitments it involves (*e.g.*, the equal reality of past, present, and future, the denial of objective temporal becoming, *etc.*). The mathematization of space and time as manifolds composed of points or instants is a product of human imagination. Oppy's merely passing reference to an indispensability argument on behalf of infinitary mathematics therefore does not suffice to justify ontological actual infinitism.

That leaves us with "the many puzzles and paradoxes" of the actual infinite, some of which were summarized in chapter 3. I, for one, am not convinced that Oppy has succeeded in blunting the force of these conundrums. What is striking about his treatment of these problem cases is that his resolution of them often involves no appeal at all to high falutin' mathematics. Rather he says that "the single most important strategy" that the actual infinitist can pursue is simply to "*outsmart*" those who present the problem cases, where "outsmart" is defined as follows. "*Outsmart,* v. To embrace the conclusion of one's opponent's *reductio ad absurdum* argument" (p. 48). I had to smile at the sheer cheekiness of this proposed strategy. It would have been more accurate, I think,

to describe this strategy as, "Just bite the bullet!" Oppy goes on to explain, "In many cases, these allegedly absurd situations are just what one ought to expect if there were . . . physical infinities." But counterfactuals of the form "If a physical infinity of such-and-such a nature were to exist, then such-and-such a situation would obtain" are not in dispute. The problem cases would not be problematic if the alleged consequences would not ensue. Rather the question is whether these consequences really are absurd.

I agree with Oppy that in many cases about the best the actual infinitist can do is just bite the bullet. Take Hilbert's Hotel. Oppy makes the point that having a hotel with an infinite number of occupied rooms does not commit one to the possibility of accommodating more guests by shifting guests about—maybe the hotel's construction hinders the guests' movements or the guests die off before their turn to move comes round. This point is trivial, however; as a *Gedankenexperiment* the hotel can be configured as we choose to stipulate. Oppy, now, has no choice but to bite the bullet: "There can, after all, be a hotel in which infinitely many new guests are accommodated, even though all the rooms are full, via the simple expedient of moving the guests in room N to room $2N$ (for all N)" (p. 53). I wonder if he would say something similar about what would happen when an infinite number of guests depart? Merely reiterating the relevant counterfactuals does nothing to allay our suspicions that Hilbert's Hotel is absurd.

There is much more to be said about Oppy's handling of other problem cases, as well as his views on a plethora of other issues discussed in the book. These discussions are often challenging because they are so tersely summarized. Nevertheless, the book has much food for thought and repays careful study and reflection.